Design of a New Position Control Algorithm for Robot Manipulators

Fernando Reyes and Edna Guevara
Benemérita Universidad Autónoma de Puebla
Grupo de Robótica de la Facultad de Ciencias de la Electrónica

Abstract: This paper describes new controllers with structure nonlinear for position of robot manipulators. They are supported by a rigorous stability analysis including the robot dynamics in the closed-loop. Besides the theoretical result, the performance of the controllers is illustrated by experimental results on a three degrees of freedom direct-drive robot manipulator.

1 Introduction

The PD control is the strategy most widely for robot manipulators because of its simplicity. It is well known that most of industrial robots are equipped with regulators PD or PID types.

The position control (also called regulation problem) of manipulator robots is one of the most relevant issues in practice of manipulator robots. The goal of position control is to move the manipulator end-effector to a fixed desired target constant in time, regardless its initial position. [1]

The popular proportional-derivative control with gravity compensation studied in [2] can be considered as a landmark in robot control, yields global asymptotically stable closed-loop for a trivial selection of proportional and derivative gains showing to be effective for position control.

In view of the simplicity and applicability of the PD controller, our principal motivation in the theoretical and practical interest of relying on control algorithms leading to global asymptotic stability of the closed-loop system. The contribution of this article is to extend the previous results on the linear PD algorithm to a larger class of controllers, we propose the design of new controllers with stability of the closed-loop dynamics.

This paper is organized as follows. Section 2 recalls the robot dynamics. In the section 3, is presented the new controllers and their stability analysis. Section 4 summarizes the main components of the experimental set-up. Section 5 contains the experimental results on direct-drive arm. Finally, we offer some conclusions in Section 6.

2 Robot dynamics

The dynamics of the robot can be obtained via the so called Lagrange-formulation. It is well known that in the absence of friction and other disturbances, the dynamics of a serial n-link rigid robot can be written as [3]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau \qquad (1)$$

where q is the $n \times 1$ vector of joint displacements, $\dot{q}$ is the $n \times 1$ vector of joint velocities. τ is the $n \times 1$ applied torques, $M(q)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix. $C(q,\dot{q})$ is the $n \times n$ matrix of centripetal and Coriolis torques, and $g(q)$ is the $n \times 1$ vector pf gravitational torques obtained as the gradient of the robot potential energy.

Although the equation of motion (1) is complex, it has several fundamental properties which can be exploited to find the stability of the controllers, such as:

Property 1. The matrix $C(q,\dot{q})$ and the time derivative $\dot{M}(q)$ of the inertia matrix satisfy:

$$\dot{q}^T \left[\frac{1}{2}\dot{M}(q) - C(q,\dot{q})\right]\dot{q} = 0 \quad \forall \quad q,\dot{q} \in \mathbb{R}^n. \qquad (2)$$

3 New PD-Type Controllers with Gravity Compensation

This section presents the new proposed controllers: P-Fractional D, P-Fractional Tanh(D), P+Senh D and P+Senh Tanh(D) respectively; and their proof of stability analysis. Now, we are in position to formulate the control problem. Consider the control schemes mentioned given by:

$$\tau = K_p \begin{bmatrix} \frac{2\,\tilde{q}_1}{1\,+\,\tilde{q}_1{}^2} \\ \vdots \\ \frac{2\,\tilde{q}_n}{1\,+\,\tilde{q}_n{}^2} \end{bmatrix} - K_v\dot{q} + g(q) \quad (3)$$

$$\tau = K_p \begin{bmatrix} \frac{2\,\tilde{q}_1}{1\,+\,\tilde{q}_1{}^2} \\ \vdots \\ \frac{2\,\tilde{q}_n}{1\,+\,\tilde{q}_n{}^2} \end{bmatrix} - K_v tanh(\dot{q}) + g(q) \quad (4)$$

$$\tau = K_p \begin{bmatrix} 2\tilde{q}_1 + senh(\tilde{q}_1) \\ \vdots \\ 2\tilde{q}_n + senh(\tilde{q}_n) \end{bmatrix} - K_v\dot{q} + g(q) \quad (5)$$

$$\tau = K_p \begin{bmatrix} 2\tilde{q}_1 + senh(\tilde{q}_1) \\ \vdots \\ 2\tilde{q}_n + senh(\tilde{q}_n) \end{bmatrix} - K_v tanh(\dot{q}) + g(q) \quad (6)$$

where $\tilde{q}_i = q_{d\,i} - q_i$, $i = 1 \cdots n$ denotes the position error, $q_{d\,i} \in \mathbb{R}$ are the desired constant joint positions, $K_p \in \mathbb{R}^{n\times n}$ is the proportional gain which is diagonal matrix and $K_v \in \mathbb{R}^{n\times n}$ is the derivative gain and also diagonal matrix; both matrixes are positive definite matrixes.

The control problem can be stated as selecting the design matrixes K_p and K_v such that the position error $\tilde{q}_i$ vanishes asymptotically to zero.

Lemma. Consider the robot dynamic model (1), let the control laws (3 to 6), then the closed-loop system is globally asymptotically stable and the position around zero is achieved.

Proof. The closed-loop system equation obtained by combining the robot model and a control scheme (3) can be written as:

$$\frac{d}{dt}\begin{bmatrix} \tilde{q} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -\dot{q} \\ M(q)^{-1}\left[K_p \begin{bmatrix} \frac{2\,\tilde{q}_1}{1\,+\,\tilde{q}_1{}^2} \\ \vdots \\ \frac{2\,\tilde{q}_n}{1\,+\,\tilde{q}_n{}^2} \end{bmatrix} - K_v\dot{q} - C(q,\dot{q})\dot{q} \right] \end{bmatrix} \quad (7)$$

which is an autonomous differential equation, and the origin of the state space is its unique equilibrium point. To carry out the stability analysis of equation (3) , we proposed the following Lyapunov function candidate based in a similarly form to the energy shaping methodology [4]:

$$V(\tilde{q},\dot{q}) = \frac{1}{2}\dot{q}^T M(q)\dot{q} + K_p \sum_{k=1}^{n} Ln(1\ +\ \tilde{q}_k{}^2) \quad (8)$$

The first term of $V(\tilde{q},\dot{q})$ is a positive definite function with respect to $\dot{q}$ because $M(q)$ is a positive definite matrix. The second term is also a positive definite function with respect to position error $\tilde{q}$ because K_p is a positive definite matrix. Therefore $V(\tilde{q},\dot{q})$ is a globally positive definite and radially unbounded function.

The time derivative of Lyapunov function candidate (8) along the trajectories of the closed-loop equation (7) is:

$$\frac{d}{dt}V(\tilde{q},\dot{q}) = \dot{q}^T K_p \begin{bmatrix} \frac{2\,\tilde{q}_1}{1\,+\,\tilde{q}_1{}^2} \\ \vdots \\ \frac{2\,\tilde{q}_n}{1\,+\,\tilde{q}_n{}^2} \end{bmatrix} - \dot{q}^T K_v\dot{q} - \dot{q}^T C(q,\dot{q})\dot{q}$$

$$+\frac{1}{2}\dot{q}^T \dot{M}(q)\dot{q} - \begin{bmatrix} \frac{2\,\tilde{q}_1}{1\,+\,\tilde{q}_1{}^2} \\ \vdots \\ \frac{2\,\tilde{q}_n}{1\,+\,\tilde{q}_n{}^2} \end{bmatrix}^T K_p\dot{q}.$$

After some algebra by using the property 1, it can be written as:

$$\frac{d}{dt}V(\tilde{q},\dot{q}) = -\dot{q}^T K_v\dot{q} \quad \leq 0.$$

which is a globally negative semi-definite function and therefore we conclude stability of the equilibrium point. In order to prove asymptotic stability we exploit the autonomous nature of the closed-loop equation (7) to apply the LaSalle invariance principle. In the region:

$$\Omega = \left\{ \begin{bmatrix} \tilde{q} \\ \dot{q} \end{bmatrix} \in \mathbb{R}^n : \dot{V}(\tilde{q}, \dot{q}) = 0 \right\}$$

the unique invariant is $\tilde{q} = 0$ and $\dot{q} = 0$. Therefore invoking the LaSalle invariance principle we conclude that the origin of the state space is globally asymptotically stable.

This proof is similar for the other three regulators. It is important to notice that the result of the time derivative of the Lyapunov function in each case is equal, then we conclude that the regulators are globally asymptotically stable.

4 Experimental Set-Up

We have designed and built at Autonomous University of Puebla an experimental system for research of robot control algorithms, it is a direct-drive robot manipulator with three degrees of freedom moving in three-dimensional space (see figure 1). The experimental robot consists of links made of 6061 aluminum actuated by brushless direct drive servo actuator from Paker Compumotor to drive the joints without gear reduction. Advantages of this type of direct-drive actuator include freedom from backslash and significantly lower joint friction compared to actuators with gear drives. The motors used in the robot manipulator are listed in the Table 1.

Joint	Model	Torque (Nm)	p/rev.
Base	DM1050A	50	1'024,000
Shoulder	DM1150A	150	1'024,000
Elbow	DM1015B	15	655,360

Table 1: Servo Actuators

The servos are operated in torque mode, so the motors act as torque source and they accept an analog voltage as a reference of torque signal. Position information is obtained from incremental encoders located on the motors. The standard backwards difference algorithm applied to the joint position measurements is used to generate the velocity signals. The manipulator workspace is a sphere with a radius of 1 m (3.28 ft).

Besides position sensors and motor drivers, it also includes a motion control board manufactured by Precision Micro Dynamics Inc., which is used to obtain the position joints. This board is mounted into a Pentium II PC (333 MHz) which is used for execution of the control algorithm. The new control algorithm has been written in the C language. It was executed at 2.5 msec. sampling rate.

Figure 1: Experimental Robot Manipulator

With reference to the robot dynamics, only the knowledge of the structure of the gravitational torque vector is required to implement the new controllers:

$$g(q) = 9.81 \begin{bmatrix} 0 \\ 3.921 sin(q_2) + 0.186 sin(q_2 + q_3) \\ 0.186 sin(q_2 + q_3) \end{bmatrix}$$

5 Experimental Results

Extensive experiments were carried out with the new controllers. Only one of them had a very good performance (P+Senh D) and it was difficult to find the values of the proportional and derivative gains for the remaining three regulators because of the nature of themselves. Nevertheless, it is presented the performance of the position errors for each controller in the next figures.

We chose the following desired joint positions: for the base 45 degrees, shoulder 45 degrees and the elbow 90 degrees. The initial positions and velocities were set to zero.

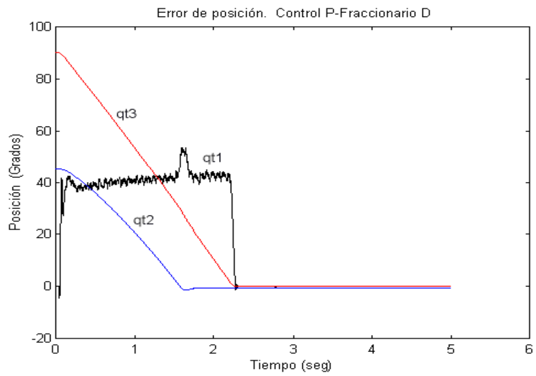

Figure 2: Position Errors using P-Fractional D regulator.

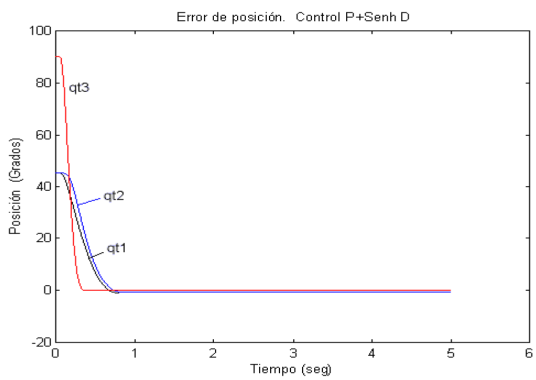

Figure 3: Position Errors using P+Senh D regulator.

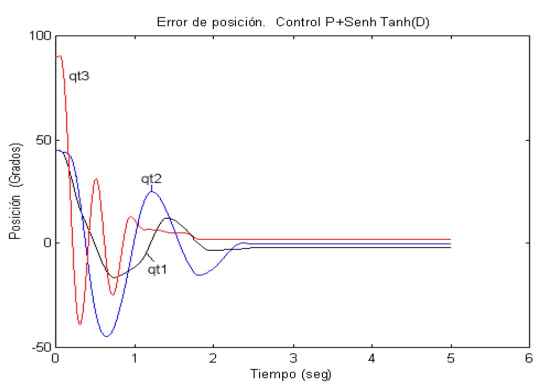

Figure 4: Position Errors using P+Senh Tanh(D) regulator.

In these figures we notice that the best performance is from P+Senh D regulator because it was able to eliminate the static friction of the robot manipulator.

6 Concluding Remarks

In this paper we have proposed 4 new controllers with gravity compensation for position control of robot manipulators, supported by a rigorous stability analysis. For stability purpose, the tuning procedure for the new controllers is trivial because it suffices to select diagonal proportional and derivative gains in order to ensure global asymptotic stability. Experiments results in real-time on a three degree-of-freedom direct drive robot system have been carried out to show the stability and performance of these regulators. For the experimental results the usefulness of the controllers can be concluded.

In these figures we notice that the P+Senh D controller has the best performance.

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